University of Manchester Spring 2010

The Standard Model of Particle Physics
Exercises II

to be handed in by 16 April 2009

1 The mass terms in the SM Lagrange density 8 points

Consider the scalar field as defined in the lecture

$$\phi = \begin{pmatrix} 0 \\ 1/\sqrt{2}\,(v + H(x)) \end{pmatrix}$$

where $H(x)$ is the physical Higgs quantum field with vanishing vacuum expectation value.

(a) Consider the Yukawa coupling term in the Lagrange density as in the lecture $\mathcal{L}_{\text{Yuk}}(x)$ and derive the mass term and the Higgs coupling term for the electron. Define the electron mass.

(b) Consider the kinetic term $(D_\lambda \phi(x))^\dagger \, (D^\lambda \phi(x))$ and calculate the mass terms and Higgs coupling terms for the physical W boson, Z boson and the photon. Define their masses.

(c) The so-called "ρ parameter" is defined as

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2(\theta_W)} \ ,$$

where θ_W is the Weinberg angle. What is the prediction for the value of the ρ parameter in the Standard Model?

2 An SU(2) triplet model — 22 points

(a) Show that the matrices,

$$T_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$
$$T_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$
$$T_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix},$$

are suitable generators of SU(2) transformations.

(b) An alternative to the usual scenario of spontaneous symmetry breaking (discussed in the notes) could have the Higgs SU(2) doublet replaced by a SU(2) triplet, i.e.

$$\phi = \begin{pmatrix} \phi^{++} \\ \phi^{+} \\ \phi^{0} \end{pmatrix}.$$

The superscripts denote the electromagnetic charge of the fields. What is the weak hypercharge of this triplet?

(c) Show that

$$\phi = \begin{pmatrix} 0 \\ 0 \\ v/\sqrt{2} \end{pmatrix}$$

is a suitable choice for the vacuum and that this choice ensures the $\mathrm{U}(1)_{\mathrm{em}}$ invariance of the ground state.

(d) If the photon is required to be massless, what is the value of the ρ parameter in this particular model?

(e) Now consider adding back a Higgs doublet, i.e. so the model has both a doublet and a triplet. Now it is possible to keep ρ close to unity providing the Higgs triplet has a sufficiently small vacuum expectation value. If ρ is constrained to lie within 0.1% of unity compute the implied constraint on the ratio of the doublet and triplet vacuum expectation values.